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RESET THRESHOLDS OF TRANSFORMATION MONOIDS¹

Abstract. Motivated by the Černý conjecture for automata, we introduce the concept of monoidal automata, which allows us the formulation of the Černý conjecture for monoids. We obtain upper bounds on the reset threshold of monoids with certain properties. In particular, we obtain a quadratic upper bound if the transformation monoid contains a primitive group of permutations and a singular of maximal rank with only one point of contraction.

Keywords: Černý conjecture, finite automaton, finite monoid, transformation monoid.

INTRODUCTION

We consider finite automata together with their transition monoids. A deterministic finite complete semiautomaton, called later automaton, is synchronizing if it admits a reset word, whose action sends all states to one state. The reset threshold of a synchronizing automaton is the minimum length of its reset words. Synchronizing automata are most famous due to the Černý conjecture from 1969, which says that the reset threshold of a synchronizing automaton with n states is at most $(n-1)^2$ [1].

In this work, we study the problem from the perspective of monoid properties. We define the reset threshold and the Černý conjecture for finite transformation monoids, which are equivalent to that for ordinary finite automata but stated apart from a particular set of generators.

The concept is not entirely new. Similar ideas appeared in many papers but in an implicit way. For example, Cayley graphs were used for groups as early as the 19th century, and in [2] they are used to investigate the Černý conjecture. In [3], there are considered some varieties of monoids and, in fact, determined their reset thresholds. From a similar standpoint, the Černý was also considered in [4], where lower and upper bounds on the reset threshold of the full transformation monoid were obtained. In [5], a theory of transformation monoids using the theory of categories was developed.

1. OUR CONTRIBUTIONS

First, the concept of the reset threshold for transformation monoids and monoidal semi-automata is introduced. Then we consider the class of transformation monoids whose digraph of singulars is strongly connected. Our main result is a quadratic upper bound on the reset threshold of a monoidal semiautomaton whose transformation monoid belongs to that class and if there is a generator that has only one point of contraction. In other words, these are transformations with the kernel type $(k, 1, \dots, 1)$, where $k \geq 2$. We also show that if $k = 2$, then such semiautomaton is completely reachable.

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